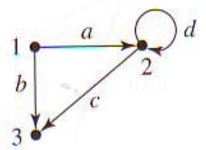
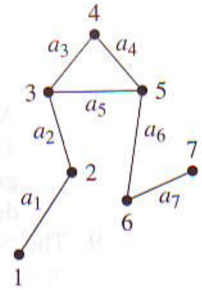
EXERCISES 5.1

1. Give the function g that is part of the formal definition of the directed graph shown.



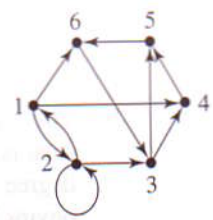
2. Answer the following questions about the accompanying graph.



   a. Is the graph simple?
   b. Is the graph complete?
   c. Is the graph connected?
   d. Can you find two paths from 3 to 6?
   e. Can you find a cycle?
   f. Can you find an arc whose removal will make the graph acyclic?
   g. Can you find an arc whose removal will make the graph not connected?

★ 3. Sketch a picture of each of the following graphs:
   a. simple graph with three nodes, each of degree 2
   b. graph with four nodes, with cycles of length 1, 2, 3, and 4
   c. noncomplete graph with four nodes, each of degree 4

★ 4. Use the directed graph in the figure to answer the following questions.



   a. Which nodes are reachable from node 3?
   b. What is the length of the shortest path from node 3 to node 6?
   c. What is a path from node 1 to node 6 of length 8?

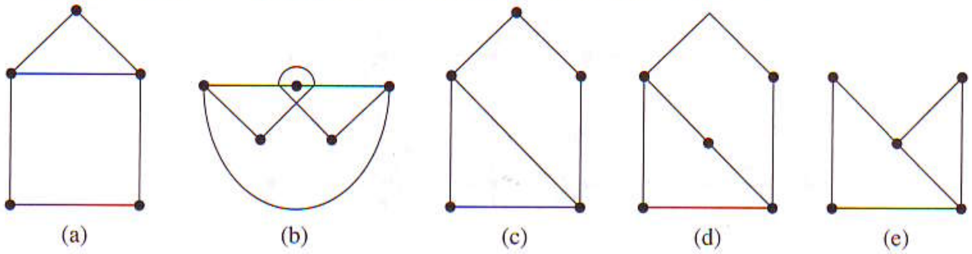
5. a. Draw K_6. b. Draw $K_{3,4}$.

6. For each of the following characteristics, draw a graph or explain why such a graph does not exist:
   a. four nodes of degree 1, 2, 3, and 4, respectively
   b. simple, four nodes of degree 1, 2, 3, and 4, respectively
   c. four nodes of degree 2, 3, 3, and 4, respectively
   d. four nodes of degree 2, 3, 3, and 3, respectively

7. For each of the following characteristics, draw a graph or explain why such a graph does not exist.
   a. simple graph with seven nodes, each of degree 3
   b. four nodes, two of degree 2 and two of degree 3
   c. three nodes of degree 0, 1, and 3, respectively
   d. complete graph with four nodes each of degree 2

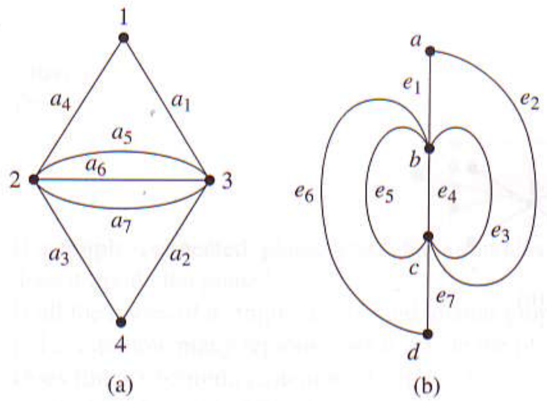
★ 8. An *acquaintanceship graph* is an undirected graph in which the nodes represent people and nodes a and b are adjacent if a and b are acquainted.
   a. The acquaintanceship graph for the IT department and the marketing department of a major corporation is an unconnected graph. What does this imply?

12. Which of the following graphs is not isomorphic to the others, and why?

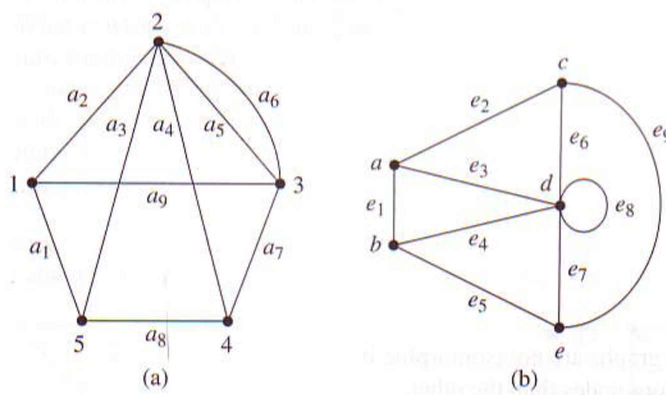


For Exercises 13–18, decide if the two graphs are isomorphic. If so, give the function or functions that establish the isomorphism; if not, explain why.

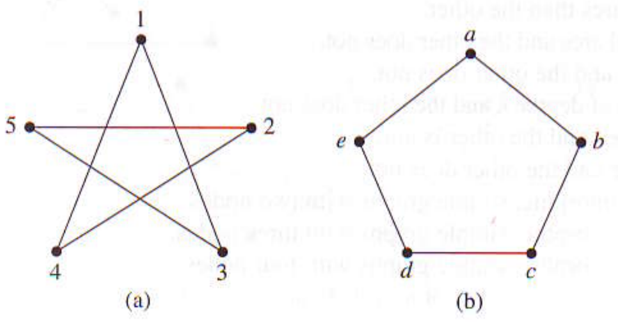
★ 13.



14.



15.



SECTION 5.2 Review

TECHNIQUES

- Construct expression trees.
- Construct array and pointer representations for binary trees.
- W Conduct preorder, inorder, and postorder traversals of a tree.

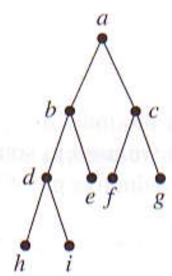
MAIN IDEAS

- Binary trees can be represented by arrays and by linked structures.
- Recursive procedures exist to systematically visit every node of a binary tree.

EXERCISES 5.2

★ 1. Sketch a picture of each of the following trees:
   a. tree with five nodes and depth 1
   b. full binary tree of depth 2
   c. tree of depth 3 where each node at depth i has $i + 1$ children

2. Answer the following questions about the accompanying graph with node a as the root.
   a. Is this a binary tree?
   b. Is it a full binary tree?
   c. Is it a complete binary tree?
   d. What is the parent of e?
   e. What is the right child of e?
   f. What is the depth of g?
   g. What is the height of the tree?



In Exercises 3–6, draw the expression tree.

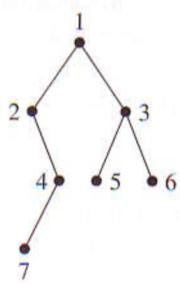
3. $[(x - 2) * 3] + (5 + 4)$

★ 4. $[(2 * x - 3 * y) + 4 * z] + 1$

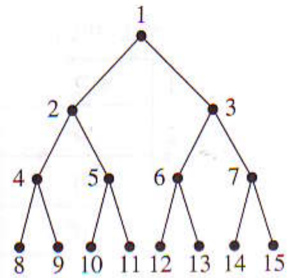
5. $1 - (2 - [3 - (4 - 5)])$

6. $[(6 \div 2) * 4] + [(1 + x) * (5 + 3)]$

★ 7. Write the left child–right child array representation for the binary tree in the figure.



8. Write the left child–right child array representation for the binary tree in the figure.



★ 9. Draw the binary tree represented by the left child–right child representation that follows. (1 is the root.)

	Left child	Right child
1	2	3
2	4	0
3	5	0
4	6	7
5	0	0
6	0	0
7	0	0

10. Draw the binary tree represented by the left child–right child representation that follows. (1 is the root.)

	Left child	Right child
1	2	0
2	3	4
3	0	0
4	5	6
5	0	0
6	0	0

11. Write the left child–right child array representation for the binary search tree that is created by processing the following list of words: "All Gaul is divided into three parts" (see Exercise 34 of Section 4.1). Also store the name of each node.

EXERCISES 5.3

★ 1. Draw the decision tree for sequential search on a list of three elements.

2. Draw the decision tree for sequential search on a list of six elements.

3. Draw the decision tree for binary search on a sorted list of seven elements. What is the depth of the tree?

4. Draw the decision tree for binary search on a sorted list of four elements. What is the depth of the tree?

★ 5. Consider a search algorithm that compares an item with the last element in a list, then the first element, then the next-to-last element, then the second element, and so on. Draw the decision tree for searching a six-element sorted list. What is the depth of the tree? Does it appear that this is an optimal algorithm in the worst case?

6. Consider a search algorithm that compares an item with an element one-third of the way through the list; based on that comparison, it then searches either the first one-third or the second two-thirds of the list. Draw the decision tree for searching a nine-element sorted list. What is the depth of the tree? Does it appear that this is an optimal algorithm in the worst case?

★ 7. a. Given the data

$$9, 5, 6, 2, 4, 7$$

construct the binary search tree. What is the depth of the tree?

b. Find the average number of comparisons done to search for an item that is known to be in the list using binary tree search on the tree of part (a). (*Hint:* Find the number of comparisons for each of the items.)

8. a. Given the data

$$g, d, r, s, b, q, c, m$$

construct the binary search tree. What is the depth of the tree?

b. Find the average number of comparisons done to search for an item that is known to be in the list using binary tree search on the tree of part (a). (*Hint:* Find the number of comparisons for each of the items.)

9. a. For a set of six data items, what is the minimum worst-case number of comparisons a search algorithm must perform?

b. Given the set of data items $\{a, d, g, i, k, s\}$, find an order in which to enter the data so that the corresponding binary search tree has the minimum depth.

10. a. For a set of nine data items, what is the minimum worst-case number of comparisons a search algorithm must perform?

b. Given the set of data items $\{4, 7, 8, 10, 12, 15, 18, 19, 21\}$, find an order in which to enter the data so that the corresponding binary search tree has the minimum depth.

★ 11. An inorder tree traversal of a binary search tree produces a listing of the tree nodes in alphabetical or numerical order. Construct a binary search tree for "To be or not to be, that is the question," and then do an inorder traversal.

12. Construct a binary search tree for "In the high and far off times the Elephant, O Best Beloved, had no trunk," and then do an inorder traversal. (See Exercise 11.)

A JPEG image file contains not only the compressed data but also the information needed to reverse the compression process (including the information to reverse the Huffman encoding). The resulting image has lost the high-frequency changes and color variations that were eliminated in the stages before the Huffman coding was applied. Parameters in the JPEG encoding process allow tradeoffs to be made between the amount of compression to be obtained and the faithfulness of the restored image to the original. Because of the nature of the algorithms used, JPEG encoding has little or no effect on black-and-white line drawings where there is no data to throw away.

A second application of Huffman encoding occurs in the VCR Plus+ remote controller device. This device simplifies VCR programming, which, as everyone knows, is understood only by eleven-year-olds. To use VCR Plus+, one enters the 1- to 8-digit PlusCode for the program to be recorded. PlusCode numbers are printed in many TV listings; a PlusCode number contains information about the date, start time, duration, and channel of the TV show to be recorded. The VCR Plus+ takes care of actually controlling the VCR. Although details of the encoding scheme are patented, a Huffman encoding is used to produce shorter PlusCode numbers for more frequently watched TV programs. Table 5.2 shows sample listings from the TV guide for Wednesday, October 10, 2001, in a midwestern city.

TABLE 5.2

Name	Network	VCR Plus+ code
Felicity	WB	9923
West Wing	NBC	39107
Cheers	Nickelodeon	490381
Tarantulas and Their Venomous Relations	Animal Planet	9789316

SECTION 5.4 Review

TECHNIQUE

W • Find Huffman codes, given a set of characters and their frequencies.

MAIN IDEA

• Given the frequency of characters in a collection of data, a binary encoding scheme can be found that minimizes the number of bits required to store the data but still allows for easy decoding.

EXERCISES 5.4

★ 1. Given the codes

Character	*a*	*e*	*i*	*o*	*u*
Encoding scheme	00	01	10	110	111

decode the sequences

a. 11011011101 b. 1000110111 c. 010101

2. Given the codes

Character	*b*	*h*	*q*	*w*	%
Encoding scheme	1000	1001	0	11	101

decode the sequences
a. 10001001101101 b. 11110 c. 01001111000

3. Given the codes

Character	*a*	*p*	*w*	(	)
Encoding scheme	001	1010	110	1111	1110

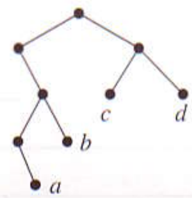
decode the sequences
a. 111110101101110001 b. 1010001110 c. 1111111100111101110

4. Given the nonprefix codes

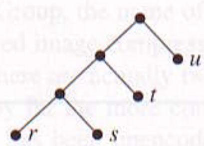
Character	1	3	5	7	9
Encoding scheme	1	111	101	10	10101

give all possible decodings of the sequence 111110101.

5. Write the Huffman codes for *a*, *b*, *c*, and *d* in the binary tree shown.



6. Write the Huffman codes for *r*, *s*, *t*, *u* in the binary tree shown.



★ 7. a. Construct the Huffman tree for the following characters and frequencies:

Character	*c*	*d*	*g*	*m*	*r*	*z*
Frequency	28	25	6	20	3	18

b. Find the Huffman codes for these characters.

8. a. Construct the Huffman tree for the following characters and frequencies:

Character	*b*	*n*	*p*	*s*	*w*
Frequency	6	32	21	14	27

b. Find the Huffman codes for these characters.

9. a. Construct the Huffman tree for the following characters and frequencies:

Character	*a*	*z*	*t*	*e*	*c*
Frequency	27	12	15	31	15

b. Find the Huffman codes for these characters.