Therefore both C(n + m, r) and $\sum_{k=0}^{r} C(n,k)C(m,r-k)$ represent the number of ways to choose r objects from the union of the two sets, and are thus equal in value.

82. $C(2) = \frac{1}{3}C(4,2) = \frac{1}{3}\frac{4!}{2!2!} = \frac{4 \cdot 3}{3 \cdot 2} = 2$

$C(3) = \frac{1}{4}C(6,3) = \frac{1}{4}\frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4}{4 \cdot 2 \cdot 3} = 5$ $\quad C(4) = \frac{1}{5}C(8,4) = \frac{1}{5}\frac{8!}{4!4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{5 \cdot 2 \cdot 3 \cdot 4} = 14$

83. a. A path sequence has length 2n and is composed of n R's and n D's in any order. There are C(2n, n) ways to select the positions for the n R's.
    b. C(2n, n) = (n + 1)C(n)

*84. 163452, 163542, 345621, 356421, 634521, 643125

85. 53124, 42531, 42315, 35142, 32541, 32154

86. 4365172

*87. 7432156

88. 3712456

89. 2761345

*90. 24589, 24678, 24679, 24689, 24789

91. Make the initial permutation the largest permutation, n...321. Then just reverse all inequalities in the body of Algorithm Permutation Generator.

92. Use the algorithm to generate all combinations of r elements from the set {1, ... , n}. For each such combination, pass it on to the algorithm to generate all permutations of r elements. That algorithm generates permutations of the elements from {1, ... , r}. Each such permutation can be used to order the way in which the elements of the combination are written out.

EXERCISES 3.5

*1. |{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}| = 8

2. 3/8

3. 1/8

*4. 2/8 = 1/4

5. 0

*6. 6·6 = 36

7. 1/36

8. 6/36 = 1/6

*9.

A	B
1	6
2	5
3	4
4	3
5	2
6	1

Probability is 6/36 = 1/6

10. (6 + 6 – 1)/36 = 11/36 (using Principle of Inclusion and Exclusion)

11.

A	B
5	6
6	5
6	6

Probability is 3/36 = 1/12.

12. An odd sum can happen by (odd, even) or (even, odd). Probability = (3·3 + 3·3)/36 = 18/36 = 1/2. (Intuitively, half the outcomes would be an odd number.)

13. 8 (this is just like Exercise 1, with finish or drop out instead of heads or tails)

14. 3/8

15. 1/8

16. 4/8

*17. C(52, 2) = 1326.

18. C(13, 2) / C(52, 2) = 78/1326 = 1/17

*19. C(39, 2) / C(52, 2) = 741/1326 = 19/34

20. C(13, 1)·C(39, 1)/C(52, 2) = 13·39 / 1326 = 13/34 (one spade, one non-spade)