14. $|A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D|$
$+ |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|$

*15. The terms in the expansion are equivalent to all of the subsets of $\{A_1, \ldots, A_n\}$ except for the empty set. Therefore there are $2^n - 1$ terms.

16. Let A = mysteries, B = spy novels, C = westerns, D = science fiction. Then $|A| = 32$, $|B| = 34$, $|C| = 18$, $|D| = 41$. Also $|A \cap B| = 17$, $|A \cap C| = 8$, $|A \cap D| = 19$, $|B \cap C| = 5$, $|B \cap D| = 20$, and $|C \cap D| = 12$. Also $|A \cap B \cap C| = 2$, $|A \cap B \cap D| = 11$, $|A \cap C \cap D| = 6$, $|B \cap C \cap D| = 5$, and $|A \cap B \cap C \cap D| = 2$.
Therefore $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D|$
$- |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D|$
$+ |B \cap C \cap D| - |A \cap B \cap C \cap D|$
$= 32 + 34 + 18 + 41 - 17 - 8 - 19 - 5 - 20 - 12 + 2 + 11 + 6 + 5 - 2 = 66$

17. 5 (Use the Pigeonhole Principle, where suits are bins, cards are items)

18. 27 (you could potentially draw 26 red cards)

*19. No - there are 13 different denominations (bins), so 12 cards could all be different.

20. 51; (you could potentially draw 50 of the same gender)

21. 3; there are two genders (bins).

22. 367

23. Yes - there are 12 bins, so more than 24 items means that at least one bin has more than 2 items.

*24. There are 3 pairs - 1 and 6, 2 and 5, 3 and 4 - that add up to 7. Each element in the set belongs to one of these pairs. Apply the Pigeonhole Principle, where the pairs are the bins, and the numbers are the items.

25. 6

26. This follows from the Pigeonhole Principle, where the n possible remainders (the numbers 0 through n - 1) are the bins.

EXERCISES 3.4

1. *a. 42 b. 6720 c. 360 d. $\frac{n!}{[n-(n-1)]!} = \frac{n!}{1!} = n!$

2. $9! = 362{,}880$

3. 14! ~ 87,178,291,000

4. 8! = 40,320; 7!·3= 15,120 (there are only 3 choices for the last character)

*5. 5! (total permutations)
3! (arrangement of the 3 R's for each distinguished permutation)
= 5·4 = 20

6. Seat one person in one chair anywhere in the circle – the position doesn't matter. Choose seats for the remaining 5 people from the remaining 5 positions relative to the first person, which gives 5! = 120 arrangements.

*7. $P(15, 3) = \frac{15!}{12!} = 15 \cdot 14 \cdot 13 = 2730$

8. a. $(26)^3$
b. P(26, 3) = 26·25·24 = 15600

9. 19!

*10. (2!)(11!)(8!) = 2(39,916,800)(40,320)

men first or women first

11. 11! ·C(12, 8)·8!

arrange women

select 8 of 12 slots between the men to fill with women

arrange men

12. Seat one person in one chair anywhere in the circle – the position doesn't matter. Choose seats for the remaining 18 people from the remaining 18 positions relative to the first person, which gives 18!

13. Seat one man in one chair anywhere in the circle – the position doesn't matter. Choose seats for the remaining 10 men in positions relative to the first man, which gives 10!. Each woman must be seated to a man's right, giving 11 locations for women of which 8 must be chosen – C(11,8). Then arrange the 8 women in these 8 chosen locations, giving 8! arrangements of women. The answer is thus 10!·C(11,8)·8!

14. *a. 120 b. 36 c. 28 d. $\frac{n!}{(n-1)!1!} = n$