b. T(0) = 1 (trivial case)
T(1) = 1 (number of ways to triangulate a 3-sided polygon)

For a fixed k, the (k + 1)-sided polygon can be triangulated in T(k – 1) ways and the (n - k + 2)-sided polygon can be triangulated T(n – k) ways, giving, by the Multiplication Principle, T(k – 1)T(n – k) triangulations. Each value for k gives a different set of triangulations, so by the Addition Principle,

$$T(n) = T(0)T(n-1) + T(1)T(n-2) + \dots T(n-1)T(0)$$
$$= \sum_{k=1}^{n} T(k-1)T(n-k)$$

c. The recurrence relations for T(n) and C(n) are identical.

EXERCISES 3.3

1. Let A = those who speak English
B = those who speak French
Then $|A \cup B| = 42$, $|A| = 35$, $|B| = 18$.
$|A \cap B| = |A| + |B| - |A \cup B| = 35 + 18 - 42 = 11$

*2. Let A = guests who drink coffee
B = guests who drink tea
Then $|A| = 13$, $|B| = 10$, and $|A \cap B| = 4$.
$|A \cup B| = |A| + |B| - |A \cap B| = 13 + 10 - 4 = 19$

3. Let A = those who bought green beans
B = those who bought beets
Then $|A| = 56$, $|B| = 38$, $|A \cap B| = 17$.
$|A \cup B| = |A| + |B| - |A \cap B| = 56 + 38 - 17 = 77$. Therefore 137 - 77 = 60 customers bought neither.

4. Let A = parts with paint defect
B = parts with packaging defect
C = parts with electronic defect
Then $|A| = 28$, $|B| - 17$, $|C| = 13$, $|A \cap B| = 6$, $|B \cap C| = 7$, $|A \cap C| = 10$.
$40 = 28 + 17 + 13 - 6 - 7 - 10 + |A \cap B \cap C|$, so $|A \cap B \cap C| = 5$; 5 parts had all three types of defect.

5. 18

*6. Let A = breath set, B = gingivitis set, C = plaque set
a. $|A \cap B \cap C| = 2$
b. $|A - C| = |A| - |A \cap C| = 12 - 6 = 6$

7. Let A = CS 120 set, B = CS 180 set, C = CS 215 set. Then $|A \cup B \cup C| = 32 + 27 + 35 - 7 - 16 - 3 + 2 = 70$. Therefore 83 - 70 = 13 students are not eligible to enroll.

8. Let A = checking account set, B = regular savings set, C = money market savings set.
   a. $|A \cap C| = 93$.
   b. $|A - (B \cup C)| = |A| - |A \cap (B \cup C)|$ by Example 29
   $= |A| - |(A \cap B) \cup (A \cap C)|$
   $= |A| - (|A \cap B| + |A \cap C|)$ by Example 28 because $(A \cap B)$ and $(A \cap C)$ are disjoint
   $= 189 - (69 + 93) = 27$

*9. Let A = auto set, B = bike set, C = motorcycle set
   a. $|B - (A \cup C)| = |B| - |B \cap (A \cup C)|$ by Example 32
   $= |B| - |(B \cap A) \cup (B \cap C)|$
   $= |B| - (|B \cap A| + |B \cap C| - |B \cap A \cap C|)$
   $= 97 - (53 + 7 - 2) = 39$

   b. $|A \cup B \cup C| = 97 + 83 + 28 - 53 - 14 - 7 + 2 = 136$, so 150 - 136 = 14 do not own any of the three.

10. From the Principle of Inclusion and Exclusion,
   $87 = 68 + 34 + 30 - 19 - 11 - 23 + |A \cap B \cap C|$, so $|A \cap B \cap C| = 8$.

11. No. Letting A, B, and C be the odor, lather, and ingredients sets, the Principle of Inclusion and Exclusion says that the union of the three sets would contain 491 people, yet only 450 were surveyed.

12. a. Let A = the integers between 1 and 100 that are multiples of 2. There are k of these, namely $1 \cdot 2, 2 \cdot 2, \dots, k \cdot 2 = 100 = 50 \cdot 2$, so $k = |A| = 50$.
   Let B = the integers between 1 and 100 that are multiples of 5. There are m of these, namely $1 \cdot 5, 2 \cdot 5, \dots, m \cdot 5 = 100 = 20*5$, so $m = |B| = 20$.
   $A \cap B$ is the set of integers that are multiples of both 2 and 5, that is, multiples of 10. There are n of these, namely $1 \cdot 10, 2 \cdot 10, \dots, n \cdot 10 = 100 = 10 \cdot 10$, so $n = |A \cap B| = 10$.
   $|A \cup B| = 50 + 20 - 10 = 60$
   b. These are the numbers that are not in $A \cup B$, of which there are $100 - 60 = 40$.

13. Let A = the integers between 1 and 1000 that are multiples of 3. There are k of these namely $1 \cdot 3, 2 \cdot 3, \dots, k \cdot 3 = 999 = 333 \cdot 3$, so $k = |A| = 333$.
   Let B = the integers between 1 and 1000 that are multiples of 7. There are m of these, namely $1 \cdot 7, 2 \cdot 7, \dots, m \cdot 7 = 994 = 142 \cdot 7$, so $m = |B| = 142$.
   $A \cap B$ is the set of integers that are multiples of both 3 and 7, that is, multiples of 21. There are n of these, namely $1 \cdot 21, 2 \cdot 21, \dots, n \cdot 21 = 987 = 47 \cdot 21$, so $n = |A \cap B| = 47$.
   $|A \cup B| = 333 + 142 - 47 = 428$
   There are $1000 - 428 = 572$ numbers that are not multiples of either 3 or 7.