

87. Let A be a countable set. Then A is finite or countably infinite. If A is finite and $B \subseteq A$, then B is finite, hence countable. If A is countably infinite, let a_1, a_2, a_3, \dots be an enumeration of A . Using this same list but eliminating elements in $A - B$ gives an enumeration of B .

*88. Let A and B be denumerable sets with enumerations $A = a_1, a_2, a_3, \dots$ and $B = b_1, b_2, b_3, \dots$. Then use the list $a_1, b_1, a_2, b_2, a_3, b_3, \dots$ and eliminate any duplicates. This will be an enumeration of $A \cup B$, which is therefore denumerable.

89. $B = \{S \mid S \text{ is a set and } S \notin S\}$. (Perhaps you think that no set S can be an element of itself, in which case B is empty. But we can still talk about set B .) Then either $B \in B$ or $B \notin B$. If $B \in B$, then B has the property of all members of B , namely $B \notin B$. Hence both $B \in B$ and $B \notin B$ are true. If $B \notin B$, then B has the property characterizing members of B , hence $B \in B$. Therefore both $B \notin B$ and $B \in B$ are true.

EXERCISES 3.2

*1. $5 \cdot 3 \cdot 2 = 30$

*2. $4 \cdot 2 \cdot 2 = 16$

3. $4 \cdot 8 \cdot 6 = 92$

4. $4^{20} \cdot 5^{10}$

5. $26^3 \cdot 10^2$

6. $52^3 \cdot 10^2$

*7. $45 \cdot 13 = 585$

8. $3 \cdot 2(A - B - D) + 2 \cdot 4(A - C - D) = 14$

*9. 10^9

10. No - the number of different codes is $10 \cdot 10 = 100$, so not every apartment has its own code.

*11. $26 \cdot 26 \cdot 26 \cdot 1 \cdot 1 = 17,576$

12. $2 \cdot 4 \cdot 4 = 32$

13. $2 \cdot 2 \cdot 2 \cdot 2$ (fill in the 4 rows of the truth table with T or F)