

87. Let  $A$  be a countable set. Then  $A$  is finite or countably infinite. If  $A$  is finite and  $B \subseteq A$ , then  $B$  is finite, hence countable. If  $A$  is countably infinite, let  $a_1, a_2, a_3, \dots$  be an enumeration of  $A$ . Using this same list but eliminating elements in  $A - B$  gives an enumeration of  $B$ .
- \*88. Let  $A$  and  $B$  be denumerable sets with enumerations  $A = a_1, a_2, a_3, \dots$  and  $B = b_1, b_2, b_3, \dots$ . Then use the list  $a_1, b_1, a_2, b_2, a_3, b_3, \dots$  and eliminate any duplicates. This will be an enumeration of  $A \cup B$ , which is therefore denumerable.
89.  $B = \{S \mid S \text{ is a set and } S \notin S\}$ . (Perhaps you think that no set  $S$  can be an element of itself, in which case  $B$  is empty. But we can still talk about set  $B$ .) Then either  $B \in B$  or  $B \notin B$ . If  $B \in B$ , then  $B$  has the property of all members of  $B$ , namely  $B \notin B$ . Hence both  $B \in B$  and  $B \notin B$  are true. If  $B \notin B$ , then  $B$  has the property characterizing members of  $B$ , hence  $B \in B$ . Therefore both  $B \notin B$  and  $B \in B$  are true.

### EXERCISES 3.2

- \*1.  $5 \cdot 3 \cdot 2 = 30$
- \*2.  $4 \cdot 2 \cdot 2 = 16$
3.  $4 \cdot 8 \cdot 6 = 92$
4.  $4^{20} \cdot 5^{10}$
5.  $26^3 \cdot 10^2$
6.  $52^3 \cdot 10^2$
- \*7.  $45 \cdot 13 = 585$
8.  $3 \cdot 2(A - B - D) + 2 \cdot 4(A - C - D) = 14$
- \*9.  $10^9$
10. No - the number of different codes is  $10 \cdot 10 = 100$ , so not every apartment has its own code.
- \*11.  $26 \cdot 26 \cdot 26 \cdot 1 \cdot 1 = 17,576$
12.  $2 \cdot 4 \cdot 4 = 32$
13.  $2 \cdot 2 \cdot 2 \cdot 2$  (fill in the 4 rows of the truth table with T or F)