EXERCISES 2.3

In Exercises 1–4, prove that the pseudocode program segment is correct by proving the loop invariant Q and evaluating Q at loop termination.

1. Function to return the value of x^2 for $x \geq 1$

```
Square(positive integer x)
Local variables:
integers i, j
  i = 1
  j = 1
  while i ≠ x do
    j = j + 2i + 1
    i = i + 1
  end while
  //j now has the value x^2
  return j
end function Square
```

Q: $j = i^2$

★ 2. Function to return the value of $x!$ for $x \geq 1$

```
Factorial(positive integer x)
Local variables:
integers i, j
  i = 2
  j = 1
  while i ≠ x + 1 do
    j = j * i
    i = i + 1
  end while
  //j now has the value x!
  return j
end function Factorial
```

Q: $j = (i - 1)!$

3. Function to return the value of x^y for $x, y \geq 1$

```
Power(positive integer x; positive integer y)
Local variables:
integers i, j
  i = 1
  j = x
  while i ≠ y do
    j = j * x
    i = i + 1
  end while
```

```
    //j now has the value x^y
    return j
end function Power
```

$$Q\text{: } j = x^i$$

4. Function to compute and write out quotient q and remainder r when x is divided by y, $x \geq 0, y \geq 1$

```
Divide(nonnegative integer x; positive integer y)
Local variables:
nonnegative integers q, r
    q = 0
    r = x
    while r ≥ y do
        q = q + 1
        r = r − y
    end while
    //q and r are now the quotient and remainder
    write("The quotient is" q "and the remainder is" r)
end function Divide
```

$$Q\text{: } x = q * y + r$$

For Exercises 5–8, use the Euclidean algorithm to find the greatest common divisor of the given numbers.

5. (2420, 70)
★ 6. (735, 90)
7. (1326, 252)
8. (1018215, 2695)

In Exercises 9–14, prove that the program segment is correct by finding and proving the appropriate loop invariant Q and evaluating Q at loop termination.

9. Function to return the value $x - y$ for $x, y \geq 0$

```
Difference(nonnegative integer x; nonnegative integer y)
Local variables:
integers i, j
    i = 0
    j = x
    while i ≠ y do
        j = j − 1
        i = i + 1
    end while
    //j now has the value x − y
    return j
end function Difference
```