SECTION 2.2 Review

TECHNIQUES

W • Use the first principle of induction in proofs.

W • Use the second principle of induction in proofs.

MAIN IDEAS

- Mathematical induction is a technique to prove properties of positive integers.
- An inductive proof need not begin with 1.
- Induction can be used to prove statements about quantities whose values are arbitrary nonnegative integers.
- The first and second principles of induction each prove the same conclusion, but one approach may be easier to use than the other in a given situation.

EXERCISES 2.2

In Exercises 1–22, use mathematical induction to prove that the statements are true for every positive integer n.

★ 1. $2 + 6 + 10 + \cdots + (4n - 2) = 2n^2$

2. $2 + 4 + 6 + \cdots + 2n = n(n + 1)$

★ 3. $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$

4. $1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$

★ 5. $4 + 10 + 16 + \cdots + (6n - 2) = n(3n + 1)$

6. $5 + 10 + 15 + \cdots + 5n = \frac{5n(n+1)}{2}$

7. $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

8. $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

★ 9. $1^2 + 3^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

10. $1^4 + 2^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

11. $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$

12. $1 + a + a^2 + \cdots + a^{n-1} = \frac{a^n - 1}{a - 1}$ for $a \neq 0, a \neq 1$

★ 13. $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

14. $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$