6. Either the weather will turn bad or we will leave on time. If the weather turns bad, then the flight may be cancelled.
★ 7. If the bill was sent today, then you will be paid tomorrow. You will be paid tomorrow.
8. The grass needs mowing and the trees need trimming. If the grass needs mowing, then we need to rake the leaves.
9. Justify each step in the proof sequence of

$$A \wedge (B \to C) \to (B \to (A \wedge C))$$

1. A
2. $B \to C$
3. B
4. C
5. $A \wedge C$

★ 10. Justify each step in the proof sequence of

$$[A \to (B \vee C)] \wedge B' \wedge C' \to A'$$

1. $A \to (B \vee C)$
2. B'
3. C'
4. $B' \wedge C'$
5. $(B \vee C)'$
6. A'

11. Justify each step in the proof sequence of

$$A' \wedge B \wedge [B \to (A \vee C)] \to C$$

1. A'
2. B
3. $B \to (A \vee C)$
4. $A \vee C$
5. $(A')' \vee C$
6. $A' \to C$
7. C

In Exercises 12–22, use propositional logic to prove that the argument is valid.

12. $A' \wedge (B \to A) \to B'$
★ 13. $(A \to B) \wedge [A \to (B \to C)] \to (A \to C)$
14. $[(C \to D) \to C] \to [(C \to D) \to D]$
★ 15. $A' \wedge (A \vee B) \to B$
16. $[A \to (B \to C)] \wedge (A \vee D') \wedge B \to (D \to C)$
17. $(A' \to B') \wedge B \wedge (A \to C) \to C$
18. $(A \to B) \wedge [B \to (C \to D)] \wedge [A \to (B \to C)] \to (A \to D)$
19. $[A \to (B \to C)] \to [B \to (A \to C)]$
20. $(A \wedge B) \to (A \to B')'$
21. $(A \to C) \wedge (C \to B') \wedge B \to A'$
22. $[A \to (B \vee C)] \wedge C' \to (A \to B)$

Use propositional logic to prove the validity of the arguments in Exercises 23–31. These will become additional derivation rules for propositional logic, summarized in Table 1.14.

TABLE 1.14

More Inference Rules		
From	Can Derive	Name/Abbreviation for Rule
$P \to Q, Q \to R$	$P \to R$ [Example 16]	Hypothetical syllogism—hs
$P \vee Q, P'$	Q [Exercise 23]	Disjunctive syllogism—ds
$P \to Q$	$Q' \to P'$ [Exercise 24]	Contraposition—cont
$Q' \to P'$	$P \to Q$ [Exercise 25]	Contraposition—cont
P	$P \wedge P$ [Exercise 26]	Self-reference—self
$P \vee P$	P [Exercise 27]	Self-reference—self
$(P \wedge Q) \to R$	$P \to (Q \to R)$ [Exercise 28]	Exportation—exp
P, P'	Q [Exercise 29]	Inconsistency—inc
$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$ [Exercise 30]	Distributive—dist
$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$ [Exercise 31]	Distributive—dist

★ 23. $(P \vee Q) \wedge P' \to Q$
24. $(P \to Q) \to (Q' \to P')$
25. $(Q' \to P') \to (P \to Q)$
26. $P \to P \wedge P$
27. $P \vee P \to P$ (*Hint:* Instead of assuming the hypothesis, begin with a version of Exercise 26; also make use of Exercise 25.)
28. $[(P \wedge Q) \to R] \to [P \to (Q \to R)]$
★ 29. $P \wedge P' \to Q$
30. $P \wedge (Q \vee R) \to (P \wedge Q) \vee (P \wedge R)$ (*Hint:* First rewrite the conclusion.)
31. $P \vee (Q \wedge R) \to (P \vee Q) \wedge (P \vee R)$ (*Hint:* Prove both $P \vee (Q \wedge R) \to (P \vee Q)$ and $P \vee (Q \wedge R) \to (P \vee R)$; for each proof, first rewrite the conclusion.)

For Exercises 32–39, use propositional logic to prove the arguments valid; you may use any of the rules in Table 1.14 or any previously proved exercise.

32. $A' \to (A \to B)$
★ 33. $(P \to Q) \wedge (P' \to Q) \to Q$
34. $(A' \to B') \wedge (A \to C) \to (B \to C)$
35. $(A' \to B) \wedge (B \to C) \wedge (C \to D) \to (A' \to D)$
36. $(A \vee B) \wedge (A \to C) \wedge (B \to C) \to C$

37. $(Y \to Z') \wedge (X' \to Y) \wedge [Y \to (X \to W)] \wedge (Y \to Z) \to (Y \to W)$

★ 38. $(A \wedge B)' \wedge (C' \wedge A)' \wedge (C \wedge B')' \to A'$

39. $(P \vee (Q \wedge R)) \wedge (R' \vee S) \wedge (S \to T') \to (T \to P)$

Using propositional logic, including the rules in Table 1.14, prove that each argument in Exercises 40–48 is valid. Use the statement letters shown.

40. If the program is efficient, it executes quickly. Either the program is efficient, or it has a bug. However, the program does not execute quickly. Therefore it has a bug. *E, Q, B*

41. If Jane is more popular, then she will be elected. If Jane is more popular, then Craig will resign. Therefore if Jane is more popular, she will be elected and Craig will resign. *J, E, C*

42. If chicken is on the menu, then don't order fish, but you should have either fish or salad. So if chicken is on the menu, have salad. *C, F, S*

43. The crop is good, but there is not enough water. If there is a lot of rain or not a lot of sun, then there is enough water. Therefore the crop is good and there is a lot of sun. *C, W, R, S*

44. If the ad is successful, then the sales volume will go up. Either the ad is successful or the store will close. The sales volume will not go up. Therefore the store will close. *A, S, C*

★ 45. Russia was a superior power, and either France was not strong or Napoleon made an error. Napoleon did not make an error, but if the army did not fail, then France was strong. Hence the army failed and Russia was a superior power. *R, F, N, A*

46. It is not the case that if electric rates go up, then usage will go down, nor is it true that either new power plants will be built or bills will not be late. Therefore usage will not go down and bills will be late. *R, U, P, B*

47. If Jose took the jewelry or Mrs. Krasov lied, then a crime was committed. Mr. Krasov was not in town. If a crime was committed, then Mr. Krasov was in town. Therefore Jose did not take the jewelry. *J, L, C, T*

48. If the birds are flying south and the leaves are turning, then it must be fall. Fall brings cold weather. The leaves are turning but the weather is not cold. Therefore the birds are not flying south. *B, L, F, C*

★ 49. a. Use a truth table to verify that $A \to (B \to C) \leftrightarrow (A \wedge B) \to C$ is a tautology.
b. Prove that $A \to (B \to C) \Leftrightarrow (A \wedge B) \to C$ by using a series of equivalences.
c. Explain how this equivalence justifies the deduction method that says: to prove $P_1 \wedge P_2 \wedge \cdots \wedge P_n \to (R \to S)$, deduce S from $P_1, P_2, \ldots, P_n$, and R.

50. The argument of the defense attorney at the beginning of this chapter was

If my client is guilty, then the knife was in the drawer. Either the knife was not in the drawer or Jason Pritchard saw the knife. If the knife was not there on October 10, it follows that Jason Pritchard didn't see the knife. Furthermore, if the knife was there on October 10, then the knife was in the drawer and also the hammer was in the barn. But we all know that the hammer was not in the barn. Therefore, ladies and gentlemen of the jury, my client is innocent.

Use propositional logic to prove that this is a valid argument.